Removal of interference from gravitational wave spectrum

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Abstract. We develop a procedure to remove interference from gravitational wave spectrum. The method is applied to the data produced by the Glasgow laser interferometer in 1996 and all the lines corresponding to the interference with the main supply are removed.

1 Introduction

In this paper we present an algorithm to remove interference from the gravitational wave (GW) spectrum. This method allows the removal of coherent lines coming from deterministic signals while keeping the intrinsic detector noise. Unlike other existing methods for removing single interference lines [2, 7], the method described here can remove the external interference without removing any gravitational wave signal that may be hidden by the interference. Therefore, it can be very useful in the search for monochromatic GW signals as those ones produced by pulsars [5, 8].

The key to this method is to determine the interference by using many harmonics of the interference signal. In the study of the data produced by the Glasgow laser interferometer in March 1996 [3], we observe in the spectrum many instrumental lines, some of them at multiples of 50 Hz. All these lines are wide, and when we compare them, we observe that their overall structure is very similar but only the scaling of the width is different (see figure 1). If we look at these lines in more detail, in smaller length Fourier transform (seconds in length), they appear as well defined small bandwidth lines which change frequency over time in the same way, while other ones remain at constant frequency. Therefore, all these lines at multiples of 50 Hz must be harmonics of a single source, for example the main supply.

In the Glasgow data, the lines at 1 KHz have a width of 5 Hz. Therefore, we can ignore these sections of the power spectrum or we can try to remove this interference in order to be able to detect GW signals hidden behind those lines. The LIGO group has also reported largely instrumental artifacts at multiples of the 60 Hz line frequency in their 40 m interferometer [1]. Therefore, this seems to be a general effect present in the different interferometers.

This electrical interference may possibly be reduced by improvements in the electronic design that will be incorporated in the next generation of detectors.

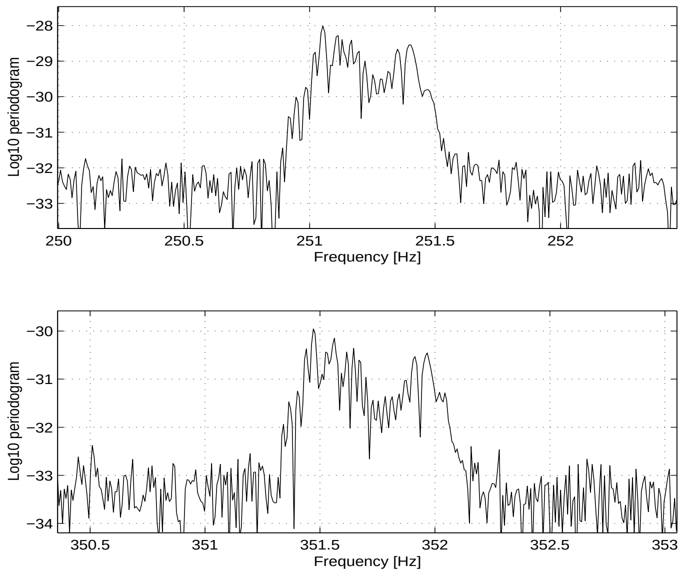

Figure 1: Comparison of the structure of the lines at 250 Hz and at 350 Hz of the power spectrum of the Glasgow data. The broad shape is due to the wandering of the incoming electricity frequency.

However, we cannot be sure that such interference will be completely absent or that other sources of interference will not manifest themselves in long-duration spectra. Indeed, the Glasgow data [3] contain other regular features of unknown origin [4]. For this reason we investigate solutions to the problem using the existing data.

We address here only the removal of coherent lines, not stochastic ones (such as those due to thermal noise). Our method requires coherence between the fundamental and several harmonics. If there is no such coherence, other methods [2, 7] can be used, but these will remove GW signals too. The method presented here is the only one we know that allows one to find real signals under the interference.

2 The basic concept

We suppose that a electrical signal at $\sim$ 50 Hz enters in the electronics of the interferometer. The instrument then transforms the input signal into a series of harmonics in a stationary way. Therefore, the data stream contains the function $y(t)$ given by

$$y(t) = \sum_n a_n m(t)^n + (a_n m(t)^n)^* \ , \tag{1}$$

where a_n are constants and $m(t)$ is nearly monochromatic around 50 Hz. The total output, $x(t)$, contains also the gravitational wave signals $s(t)$ and the detector noise $n(t)$

$$x(t) = y(t) + n(t) + s(t) \ . \tag{2}$$

We know that the data recorded is band-limited since an anti-aliasing filter was applied to the data before it was sampled. Therefore, the function $y(t)$ must be also band-limited, and hence, the sum can be limited to the first 39 harmonics. This number is given by the Nyquist frequency that is related to the sampling frequency of the experiment ($n_{max} = f_{Nyquist}/50 \text{ Hz} - 1$).

Our purpose is to construct a function

$$h(t) = \sum_{n=1}^{39} \rho_n M(t)^n + (\rho_n M(t)^n)^* \ , \tag{3}$$

similar to $y(t)$. Thus, we have to determine the complex function $M(t)$ and all the parameters ρ_n. Notice that from the experimental data we do not independently know the value of the input signal $m(t)$.

3 The algorithm

In this section we present the algorithm to remove interference from the gravitational wave spectrum.

As we pointed out in the previous section, we assume that the data produced by the interferometer is just the sum of the interference plus the detector noise and possible GW signals that we will not consider here

$$x(t) = y(t) + n(t) \ , \tag{4}$$

where $y(t)$ is given by Eq. (1). Therefore, the Fourier transform of the data $\tilde{x}(\nu)$ is simply given by

$$\tilde{x}(\nu) = \tilde{y}(\nu) + \tilde{n}(\nu) \ . \tag{5}$$

First, we select a set of harmonics (in our case the odd harmonics were strong, i.e., $k = [3, 5, 7, 9, \ldots]$), or all of them if we prefer. The idea is to construct the function $M(t)$ getting the maximum information inbeded in the harmonics considered. For each of these harmonics, we determine the initial and final frequency of the line (ν_{ik}, ν_{fk}) and we define a set of functions $\tilde{z}_k(\nu)$ in the frequency domain as

$$\tilde{z}_k(\nu) = \begin{cases} \tilde{x}(\nu) & \nu_{ik} < \nu < \nu_{fk} \\ 0 & \text{elsewhere} \ . \end{cases} \tag{6}$$

Comparing Eq. (6) with (1) and (5) we have

$$\tilde{z}_k(\nu) = a_k \widetilde{m^k} + \tilde{n}_k(\nu) \ , \tag{7}$$

where

$$\tilde{n}_k(\nu) = \begin{cases} \tilde{n}(\nu) & \nu_{ik} < \nu < \nu_{fk} \\ 0 & \text{elsewhere} \ , \end{cases} \tag{8}$$

is considered to be a zero-mean random complex noise.

Then, we calculate their inverse Fourier transform

$$z_k(t) = a_k m(t)^k + n_k(t) \ , \tag{9}$$

and we define

$$B_k(t) \equiv [z_k(t)]^{1/k} = (a_k)^{1/k} m(t) \beta_k(t) \ , \tag{10}$$

where

$$\beta_k(t) = \left[1 + \frac{n_k(t)}{a_k m(t)^k} \right]^{1/k} \ . \tag{11}$$

All these function $\{B_k(t)\}$ are almost monochromatic at ~ 50 Hz, but they have different amplitude and phase. In order to compare them, we construct another set of functions

$$b_k(t) = \alpha m(t) \beta_k(t) \ , \tag{12}$$

by performing the operation

$$b_k(t) = \Gamma_k B_k(t) \ , \qquad \Gamma_k = \sum_j B_1(j\Delta t) B_k(j\Delta t)^* \Big/ \sum_j |B_k(j\Delta t)|^2 \ . \tag{13}$$

These new functions $\{b_k(t)\}$ form a set of random variables −functions of time− and they all have the same mean value

$$\langle b_i \rangle = \alpha m(t) \ . \tag{14}$$

Now, we want to construct $M(t)$ as a function of all $\{b_k(t)\}$, in such a way that it has the same mean and minimum variance. If we assume the function $M(t)$ to be linear with $\{b_k(t)\}$, the statistically the best is

$$M(t) = \left(\sum_k \frac{b_k(t)}{\mathrm{Var}[\beta_k(t)]} \right) \Big/ \left(\sum_k \frac{1}{\mathrm{Var}[\beta_k(t)]} \right) \ , \tag{15}$$

where

$$\mathrm{Var}[\beta_k(t)] = \frac{1}{k^2} \frac{\langle n_k(t) n_k(t)^* \rangle}{|a_k m(t)^k|^2} + \text{corrections} \ , \tag{16}$$

$$a_k m(t)^k \approx z_k(t) \ , \tag{17}$$

$$\langle n_k(t) n_k(t)^* \rangle = \int d\nu \int d\nu' \langle \tilde{n}_k(\nu) \tilde{n}_k(\nu')^* \rangle e^{2\pi i(\nu - \nu')t} \ . \tag{18}$$

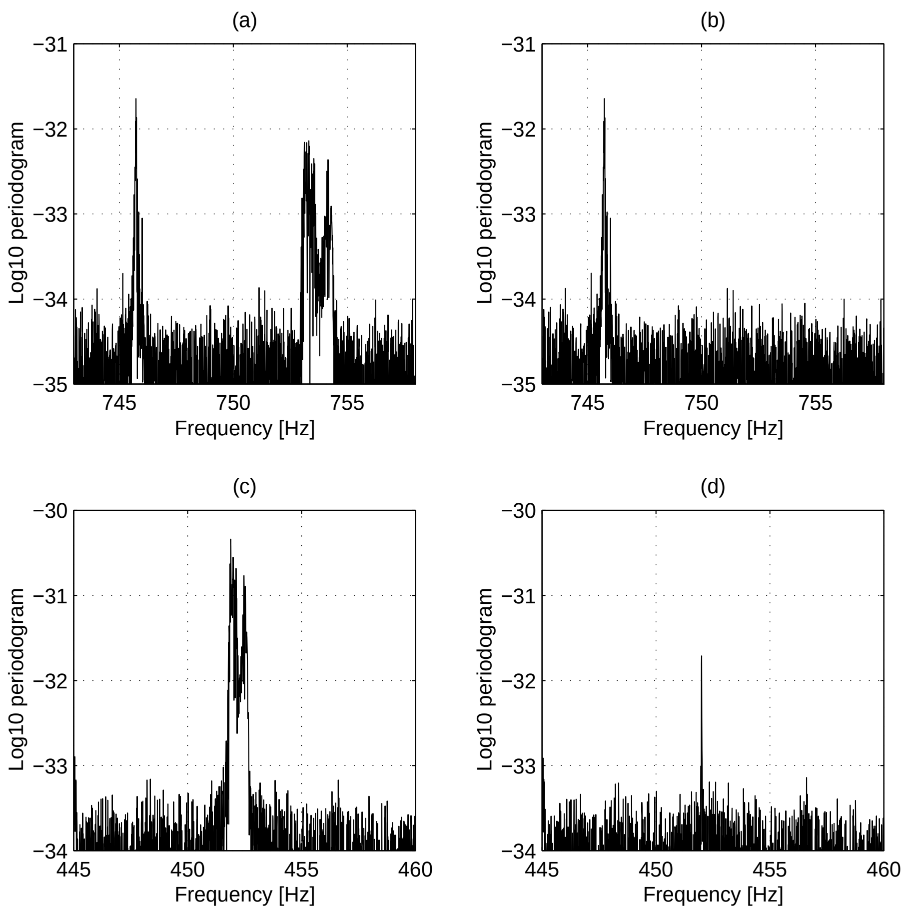

Figure 2: Decimal logarithm of the periodogram of 128 blocks (approximately 2 minutes) of the Glasgow data. (a) One of the harmonics near 754 Hz. (b) The same data after the removal of the interference as described in the text. (c) The same experimental data with an artificial signal added at 452 Hz. (d) The data in (c) after the removal of the interference, revealing that the signal remains detectable. Its amplitude is hardly changed by removing the interference.

If we assume the noise to be stationary (i.e., $\langle \tilde{n}(\nu)\tilde{n}(\nu')^* \rangle = S(\nu)\delta(\nu-\nu')$), the previous equation becomes

$$\langle n_k(t)n_k(t)^* \rangle = \int d\nu \int d\nu' \, S_k(\nu)\delta(\nu-\nu')e^{2\pi i(\nu-\nu')t} = \int_{\nu_{ik}}^{\nu_{fk}} S(\nu)d\nu \ , \quad (19)$$

where $S(\nu)$ is the power spectral density of the noise.

Finally, it only remains to determine the parameters ρ_n in Eq. (3) that we obtain by performing the operation

$$\rho_n = \sum_j x(j\Delta t)M^n(j\Delta t)^* \Big/ \sum_j |M^n(j\Delta t)|^2 \ . \quad (20)$$

We have applied this method to the data taken from the Glasgow laser interferometer in 1996 and we have succeeded in removing the electrical interference. The same method has also been applied to the true experimental data with an external simulated signal at 452 Hz, that remains hidden due to its weakness, and we have succeeded in removing the electrical interference while keeping the signal present in the data, obtaining a clear outstanding peak over the noise level (see figure 2). This will be described in detail elsewhere [6].

This method can also be applied to any other kind of interference and it may have more applications, not only for the detection of GW radiation, but also, for example, in radioastronomy.

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